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Author 1 [1,*] **and Author 2** [2]

[1] *Department of XXXXX, University of YYYYY*

[*] ***Lecturer.*** *Email: NNNNN*

[2] *Department of WWWWW, University of ZZZZZ*

Email: MMMMM

Abstract

There are two interesting methods, in the literature, for solving fuzzy linear programming problems in which the elements of coefficient matrix of the constraints are represented by real numbers and rest of the parameters are represented by symmetric trapezoidal fuzzy numbers. The first method, named as fuzzy primal simplex method, assumes an initial primal basic feasible solution is at hand. The second method, named as fuzzy dual simplex method, assumes an initial dual basic feasible solution is at hand. In this paper, the shortcomings of these methods are pointed out and to overcome these shortcomings, a new method is proposed to determine the fuzzy optimal solution of such fuzzy problems.

Keywords: Linear programming, symmetric trapezoidal fuzzy numbers, fuzzy simplex methods.

1. Introduction

The fuzzy set theory is being applied massively in many fields these days. One of these is linear programming problems. However, in most practical applications of linear programming the possible values of the parameters required in the modeling of the problem are provided either by a decision maker subjectively or a statistical inference from the past data due to which there exists some uncertainty. In order to reflect this uncertainty, the model of the problem is often constructed with fuzzy data [5]. Since the fuzziness may appear in a linear programming problem in many ways, the definition of fuzzy linear programming problem is not unique. In this paper, we consider a kind of fuzzy linear programming problem in which the elements of coefficient matrix of the constraints are represented by real numbers and rest of the parameters are represented by symmetric trapezoidal fuzzy numbers. We name this kind of fuzzy linear programming problem as semi fully fuzzy linear programming (SFFLP) problem.

Ganesan and Veeramani [2] defined SFFLP problems for the first time and then extend the primal simplex method in crisp environment for finding the fuzzy optimal solution. Their method begins with a fuzzy primal basic feasible solution for SFFLP problem and moves to an optimal basis by walking through a sequence of fuzzy primal feasible bases of SFFLP problem. All the bases with the possible exception of the optimal basis found in fuzzy primal simplex method don't satisfy the optimality criteria for SFFLP problem. Also their method has no efficient when a primal basic feasible solution is not at hand.

Nasseri and Mahdavi-Amiri [4] and Nasseri et al. [3] developed the concept of duality for the SFFLP problem proved the duality results in fuzzy sense. Based on these results, Ebrahimnejad and Nasseri [1] generalized the dual simplex method in crisp environment for obtaining the fuzzy optimal solution. Their method begins with a basic dual solution and proceeds by pivoting through a series of dual basic fuzzy solution until the associated complementary primal basic solution is feasible. However, the fuzzy dual simplex method needs to an initial dual basic feasible solution. Here, we develop the fuzzified version of conventional primal-dual method of linear programming problems that any dual feasible solution, whether basic or not, is adequate to initiate this method.

The rest of this paper is organized as follows: In Section 2, we review the basic definitions and results on fuzzy sets and related topics. Section 3 gives the definition of SFFLP problem proposed by Ganesan and

Veeramani. We give a new method for solving SFFLP problem in Section 4. The conclusions are discussed in Section 5.

2. Preliminaries

In this section, some basic definitions, arithmetic operations of fuzzy numbers and an existing ranking approach for comparing fuzzy numbers are presented [1,5].

2.1 Basic definitions

Definition 1: Let $\tilde{a}$ be a fuzzy set on R. Then, $\tilde{a}$ is a fuzzy number if and only if there exist a closed interval $[m,n] \neq \phi$ such that

$$\mu_{\tilde{a}}(x) = \begin{cases} L(x) & x \in (-\infty, m] \\ 1 & x \in [m,n] \\ R(x) & x \in [n,\infty) \end{cases} \tag{1}$$

where $L:(-\infty,m] \to [0,1]$ is monotonic increasing, continuous from the right and $L(x)=0$ for $x \in (-\infty, w_1], w_1 < m$ and $R:[n,\infty) \to [0,1]$ is monotonic decreasing, continuous from the left and $R(x)=0$ for $x \in [w_2, \infty], w_2 > n$.

As the set of fuzzy numbers is rather large and their arithmetic is in general computationally expensive based on Zadeh's extension principle, it is imperative to define and select a few special types of fuzzy numbers to be used for real life applications. Some such special types of fuzzy numbers and their arithmetic are being discussed here which will be used extensively in later section on fuzzy linear programming problems.

Definition 2: A fuzzy set $\tilde{a} = (a_1, a_2, \alpha_1, \alpha_2)$ is called a trapezoidal fuzzy numbers if its membership function is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-(a_1-\alpha_1)}{\alpha_1} & a_1-\alpha_1 \le x \le a_1 \\ 1 & a_1 \le x \le a_2 \\ \dfrac{(a_2+\alpha_2)-x}{\alpha_2} & a_2 \le x \le a_2+\alpha_2 \end{cases} \tag{2}$$

Remark 1: If $\alpha_1 = \alpha_2 = \alpha$ in the trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, \alpha_1, \alpha_2)$, we obtain a symmetric trapezoidal fuzzy number, and we denote it as $\tilde{a} = (a_1, a_2, \alpha, \alpha)$.

2.2 Arithmetic on fuzzy numbers

Let $\tilde{a} = (a_1, a_2, \alpha, \alpha)$ and $\tilde{b} = (b_1, b_2, \beta, \beta)$ be two symmetric trapezoidal fuzzy numbers. Then, arithmetic operation on these fuzzy numbers can be defined as follows:

Addition: $\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, \alpha + \beta, \alpha + \beta)$

Subtraction: $\tilde{a} - \tilde{b} = (a_1 - b_2, a_2 - b_1, \alpha + \beta, \alpha + \beta)$

Multiplication: $\tilde{a}_1\tilde{a}_2 = \left((\frac{a_1^L + a_1^U}{2})(\frac{a_2^L + a_2^U}{2}) - t, (\frac{a_1^L + a_1^U}{2})(\frac{a_2^L + a_2^U}{2}) + t, \left| a_1^U \alpha_2 + a_2^U \alpha_1 \right|, \left| a_1^U \alpha_2 + a_2^U \alpha_1 \right| \right),$

where,

$$t = \frac{t_2 - t_1}{2},\ t_1 = \min \left\{ a_1^L a_2^L, a_1^L a_2^U, a_1^U a_2^L, a_1^U a_2^U \right\},\ t_1 = \max \left\{ a_1^L a_2^L, a_1^L a_2^U, a_1^U a_2^L, a_1^U a_2^U \right\}$$

For the above definition, it can be seen that

$\lambda \geq 0,\ \lambda \in R, \lambda\tilde{a} = (\lambda a_1, \lambda a_2, \lambda\alpha, \lambda\alpha)$

$\lambda < 0,\ \lambda \in R,\ \lambda\tilde{a} = (\lambda a_2, \lambda a_1, -\lambda\alpha, -\lambda\alpha)$

It should to be noted that depending upon the need, one can also use a smaller t in the definition of multiplication involving symmetric trapezoidal fuzzy numbers.

2.3 Order on fuzzy numbers

Ranking of fuzzy numbers is an important issue in the study of fuzzy set theory. Ranking procedures are also useful in various applications and one of them will be in the study of fuzzy mathematical programming in later sections. There are numerous methods proposed in the literature for the ranking of fuzzy numbers, some of them seem to be good in a particular context but not in general. Here, we describe only a simple method for the ordering of fuzzy numbers.

Let $\tilde{a} = (a_1, a_2, \alpha, \alpha)$ and $\tilde{b} = (b_1, b_2, \beta, \beta)$ be two symmetric trapezoidal fuzzy numbers. Define the relations $\precsim$ and $\simeq$ as given below:

$\tilde{a} \precsim \tilde{b}$ if and only if

i) $\frac{(a_1 - \alpha) + (a_2 + \alpha)}{2} < \frac{(b_1 - \beta) + (b_2 + \beta)}{2}$, in this case we may write $\tilde{a} \prec \tilde{b}$.

ii) Or $\frac{a_1 + a_2}{2} = \frac{b_1 + b_2}{2}, b_2 < b_1$ and $a_2 < b_2$.

iii) Or $\frac{a_1 + a_2}{2} = \frac{b_1 + b_2}{2}, b_1 = a_1,\ a_2 = b_2$ and $\alpha \leq \beta$.

Note that in cases (ii) and (iii), we also write $\tilde{a} \approx \tilde{b}$ and say that $\tilde{a}$ and $\tilde{b}$ are equivalent.

3. Semi fully fuzzy linear programming problem

Ganesan and Veeramani [2] introduced a new type fuzzy linear programming problems in which the elements of coefficient matrix of the constraints are represented by real numbers and rest of the parameters are represented by symmetric trapezoidal fuzzy numbers. We named these kinds of problems as semi fully fuzzy linear programming (SFFLP) problems.

Here we first review two existing methods for solving SFFLP problem and then point out the shortcomings of these methods.

A SFFLP problem is defined as follows:

$$\begin{aligned} \min\ & \tilde{z} \approx c\tilde{x} \\ s.t.\ & A\tilde{x} \succsim \tilde{b} \\ & \tilde{x} \succsim \tilde{0} \end{aligned} \tag{3}$$

Definition 3: Suppose $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)$ solves $A\tilde{x} \approx \tilde{b}$. If all $\tilde{x}_j \cong (-\tilde{x}_j, \tilde{x}_j, \alpha_j, \alpha_j)$ for some $\tilde{x}_j, \alpha_j \geq 0$, then $\tilde{x}$ is said to be a fuzzy basic feasible solution. If $\tilde{x}_j \ncong (-\tilde{x}_j, \tilde{x}_j, \alpha_j, \alpha_j)$ for some $\_\tilde{x}_j, \alpha_j \geq 0$, then $\tilde{x}$ have some non-zero components, say $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_k$ $1 \leq k \leq m$. Then $A\tilde{x} \approx \tilde{b}$ can be written as:

$$a_1\tilde{x}_1 + a_2\tilde{x}_2 + \cdots + a_k\tilde{x}_k + a_{k+1}(-\tilde{x}_{k+1}, \tilde{x}_{k+1}, \alpha_{k+1}, \alpha_{k+1}) + \cdots + a_{n1}(-\tilde{x}_{n1}, \tilde{x}_{n1}, \alpha_{n1}, \alpha_n) \cong \tilde{b}$$

If the columns $a_1, a_2, \cdots, a_k$ corresponding to these non-zero components $\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_k$ are linear independent, then $\tilde{x}$ is said to be fuzzy basic feasible solution.

Remark 2: Given a system of m simultaneous fuzzy linear equations involving symmetric trapezoidal fuzzy numbers in n unknowns $A\tilde{x} \approx \tilde{b}$, where $rank(A) = m$. Let B be any $m \times m$ matrix formed by m linearly independent of A. In this case $\tilde{x} \cong (\tilde{x}_B, \tilde{x}_N) \cong (B^{-1}\tilde{b}, \tilde{0})$ is a fuzzy basic feasible solution.

Suppose a fuzzy basic feasible solution of (3) with basis B is at hand. Let y_j and $\tilde{w}$ be the solutions to $By_j = a_j$ and $\tilde{w}B = \tilde{c}_B$, respectively. Define $\tilde{z}_j \approx \tilde{c}_B y_j = \tilde{c}_B B^{-1} a_j$. Now, we are in a position to define dual of semi fully fuzzy linear programming problems and then propose a new method for finding its fuzzy optimal solution.

Definition 4: Dual of SFFLP problem (3) is defined as follows:

$$\begin{aligned} \max\ & \tilde{u} \approx y\tilde{b} \\ s.t.\ & yA \leq c \\ & y \geq 0 \end{aligned} \tag{4}$$

The method proposed by Ganesan and Veeramani in [2], starts with a fuzzy basic feasible solution for SFFLP and moves to an optimal basis by walking through a sequence of fuzzy feasible bases of FLP. All the bases with the possible exception of the optimal basis obtained in fuzzy primal simplex algorithm don't satisfy the optimality criteria for SFFLP or feasibility condition for dual of SFFLP. Since their method has no efficient when a primal basic feasible solution is not at hand thus, Ebrahimnejad and Nasseri [1] developed a new dual simplex algorithm to overcome this shortcoming by using the duality results which have been proposed by Nasseri and Mahdavi-Amiri [4] and Nasseri et al. [3]. This algorithm starts with a dual basic feasible solution, but primal basic infeasible solution and walks to an optimal solution by moving among adjacent dual basic feasible solutions. However, the dual simplex method for solving FLP problem needs to an initial dual basic feasible solution. Here, we develop the fuzzified version of conventional primal-dual method of linear programming problems that any dual feasible solution, whether basic or not, is adequate to initiate this method.

Initialization Step:

Choose a fuzzy vector $\tilde{\tilde{y}}$ such that $\tilde{z}_j - \tilde{c}_j \leq \tilde{0}$, for all j.

Main Step:

Let $\Omega = \left\{ j : \tilde{\tilde{y}}_j - \tilde{c}_j \cong \tilde{0} \right\}$. Solve the following restricted fuzzy primal problem.

$$
\begin{aligned}
\min \tilde{x}^0 &= \sum_{j \in \Omega} \tilde{0}\tilde{x}_j + \tilde{1}\tilde{x}_a \\
&\sum_{j \in \Omega} a_j \tilde{x}_j + I\tilde{x}_a \cong \tilde{b} \\
&\tilde{x}_j \tilde{\geq} \tilde{0} \quad for\,all \;\; j \in \Omega \\
&\tilde{x}_a \tilde{\geq} \tilde{0}
\end{aligned} \tag{5}
$$

If $\tilde{x}^0 \cong \tilde{0}$ then stop; the current solution is optimal. Else suppose $\tilde{\bar{v}}$ be the optimal solution of dual of problem (4).

(1) If $\tilde{\bar{y}} a_j \leq \tilde{0}$ then stop; the SFFLP problem is infeasible. Else let

$$\alpha = \alpha_k = -\frac{\bar{y}_{1k} + \bar{y}_{2k}}{\bar{v}_{1k} + \bar{v}_{2k}} = \min_j \left\{ \frac{\bar{y}_{1j} + \bar{y}_{2j}}{\bar{v}_{1j} + \bar{v}_{2j}} \,\middle|\, \bar{v}_{1j} + \bar{v}_{2j} > 0 \right\}$$

(2) Replace $\tilde{\bar{y}}$ by $\tilde{\bar{y}} + \alpha \tilde{\bar{v}}$ and go to step 1.

4. Conclusion

In this paper, we introduced a fuzzy primal-dual algorithm for solving the SFFLP problems directly without converting them to crisp linear programming problems, based on the interesting results which have been established by Ganesan and Veeramani [2].This approach can be expected to be efficient if an initial dual fuzzy solution can be computed readily.

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